Exam Quantum Physics 2

Date	9 July 2014
Room	A. Jacobshal 01
Time	9:00 - 12:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- Raise your hand for more paper
- You are allowed to use the book "Introduction to Quantum Mechanics" by Griffiths
- You are *not* allowed to use print-outs, notes or other books
- The weights of the exercises are given below
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Weighting

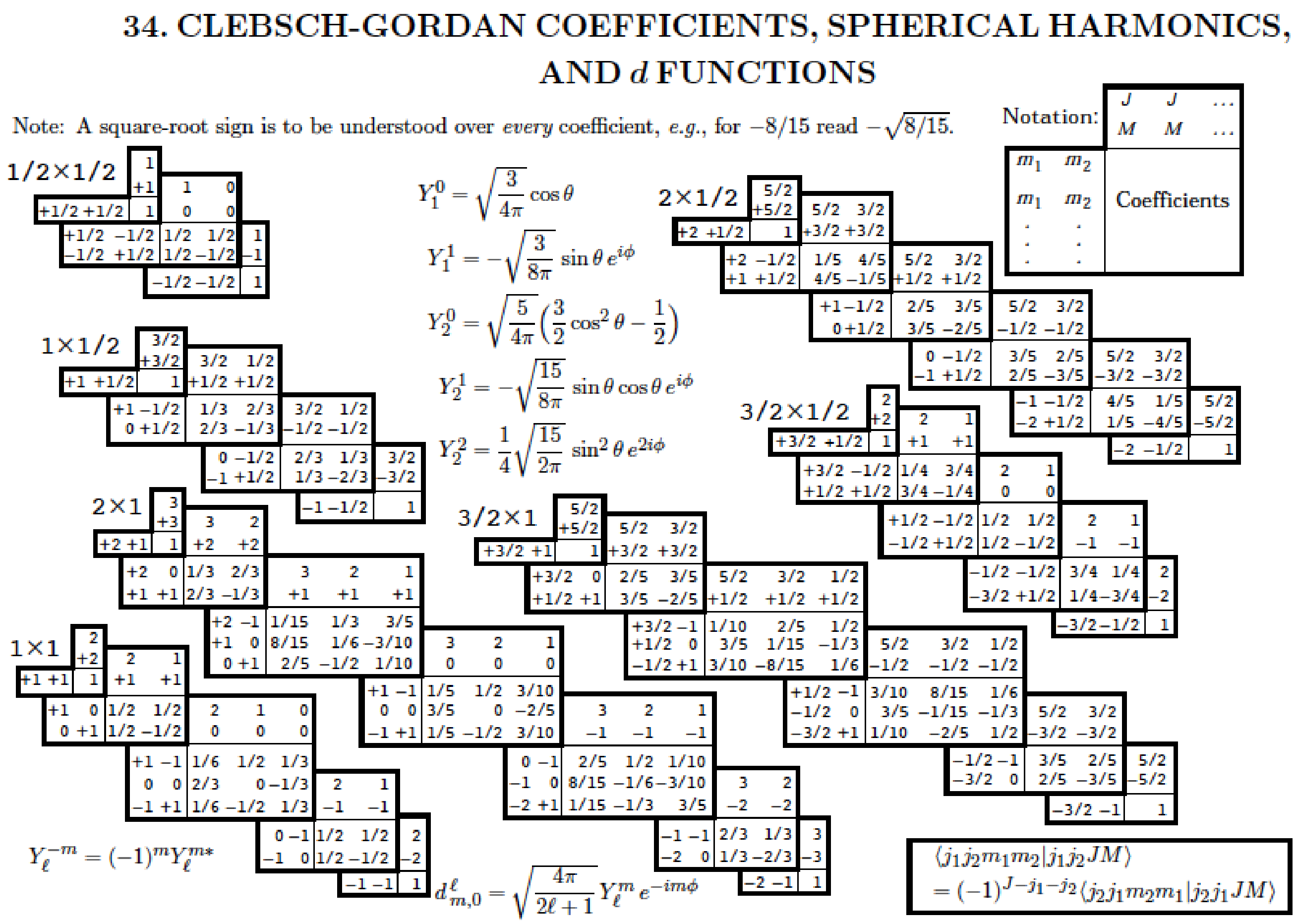
1a)	6	2a)	6	3a)	15	4)	15
1b)	9	2b)	6	3b)	6		
1c)	9	2c)	9				
1d)	9						

$$\text{Result} = \frac{\sum \text{points}}{10} + 1$$

Exercise 1

(a) Write down the general form of the Clebsch-Gordan decomposition of states $|j_1, j_2; j, m\rangle$ into $|j_1, j_2; m_1, m_2\rangle$ and vice versa. Specify explicitly the summations by indicating the variables that are summed over and their ranges.

(b) Use the table below to write down the Clebsch-Gordan decompositions of the states $|l_1, m_1\rangle|l_2, m_2\rangle = |1, 1\rangle|1, -1\rangle$ and $|l_1, m_1\rangle|l_2, m_2\rangle = |1, 0\rangle|1, -1\rangle$, and relate these two decompositions by using L_- on the first of these states.



(c) Show that $[L_i, p_j] = i\hbar \sum_k \epsilon_{ijk} p_k$ and use it to compute $[\vec{L}^2, p_z]$. Conclude whether eigenstates of p_z have definite angular momentum l or not.

(d) Explain why the eigenstates of Helium are either symmetric or antisymmetric in the interchange of the positions of the two electrons. Write down the unperturbed wave functions if one views the electrostatic repulsion between the electrons as a perturbation.

Exercise 2

Consider a two-dimensional square well potential:

$$V(x,y) = \begin{cases} 0 & \text{for } 0 \le x \le a \text{ and } 0 \le y \le a \\ \infty & \text{elsewhere} \end{cases}$$

(a) The first excited state of the system is degenerate. Give its energy and the explicit expressions for the corresponding wave functions.

Next introduce the perturbation:

$$H'(x,y) = \begin{cases} kx & \text{for } 0 \le x \le a \text{ and } 0 \le y \le a \text{ and } k > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(b) Explain using symmetry arguments why in this case one does not have to consider off-diagonal elements when using degenerate perturbation theory.

(c) Calculate in first order perturbation theory the correction(s) to the energy level of the first excited state. You may make use of the following integrals:

$$\int_0^a x \sin^2(n\pi x/a)dx = \frac{a^2}{4} \ , \quad \int_0^a x \sin(\pi x/a)\sin(2\pi x/a)dx = -\frac{8a^2}{9\pi^2},$$

where n is an integer.

Exercise 3

Consider the delta-function potential

$$V(x) = -\alpha\delta(x),$$

where α is a positive constant. This potential admits one bound state solution with energy $E = -m\alpha^2/(2\hbar^2)$.

(a) Determine the best approximation to the energy of this bound state that one can achieve with the variational method using the Lorentzian trial wave function

$$\psi_T(x) = A\,\frac{1}{x^2+a^2}.$$

You may make use of the following integrals:

$$\int_{-\infty}^{\infty} \frac{1}{(x^2+a^2)^2}dx = \frac{\pi}{2|a|^3} \ , \quad \int_{-\infty}^{\infty} \frac{1}{(x^2+a^2)^3}dx = \frac{3\pi}{8|a|^5},$$

$$\int_{-\infty}^{\infty} \frac{1}{(x^2+a^2)^4}dx = \frac{5\pi}{16|a|^7} \ , \quad \int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)^4}dx = \frac{\pi}{16|a|^5}.$$

(b) The Lorentzian trial wave function does not satisfy all properties of the true wave function at $x = 0$. Write down a trial wave function that does satisfy all requirements at $x = 0$. Draw a picture of the potential and this wave function.

Exercise 4

Consider the Hamiltonian $H = H_0 + H'(t)$, where H_0 is time independent and H' is a time-dependent perturbation. Consider the particular case of a two-level system consisting of states a and b, and $H' = \theta(t)e^{-t/\tau}V$, with V an $\vec{r}$-dependent, t-independent potential and τ is a fixed time scale. Derive in first-order time-dependent perturbation theory the probability that the system is in state b in the limit $t \to \infty$, assuming it is in state a for $t < 0$.